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THEORETICAL STUDIES OF THREE DIMENSIONAL TRANSONIC FLOW THROUGH A COMPRESSOR BLADE ROW

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This document contains a summary of efforts to develop a numerical method for calculating the inviscid, three-dimensional, transonic flow through a compressor blade row. An implicit time-marching algorithm was modified to include periodicity, boundary, and far-field conditions appropriate to internal flow. The computational grid and boundary-conforming coordinates were calculated by a conformal mapping technique, and metrics of the transformation were evaluated from analytic formulas. Calculations of the flow over a

cont
20. two-dimensional cascade using a coarse grid, showed that metric singularities near the trailing edge produced large oscillations in the solution. Several approaches for alleviating this problem are discussed.

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Section 1
INTRODUCTION

Three dimensional flow effects play an important role in the performance of axial-flow fans and compressors that operate at transonic speeds. The coupling between transonic and three-dimensional effects limits the applicability of the two-dimensional analysis methods that have been in use for some years.

Under a previous contract with Calspan Corporation,* AFOSR sponsored a study of the applicability of finite-difference computational methods to this problem.[1-5] That study led to the development of a computer program which

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*Air Force Office of Scientific Research Contract No. F44620-24-C-0059.

1. Rae, W.J., "Nonlinear Small-Disturbance Equations for Three-Dimensional Transonic Flow Through a Compressor Blade Row", Calspan Report No. AB-5487-A-1, AFOSR-TR-76-1082, AD-A031234, (August 1976).

2. Rae, W.J., "Relaxation Solutions for Three-Dimensional Transonic Flow Through a Compressor Blade Row, in the Nonlinear Small-Disturbance Approximation", Calspan Report No. AB-5487-A-2, AFOSR-TR-76-1081, AD-A032553, (August 1976).

3. Rae, W.J., "Finite-Difference Calculations of Three-Dimensional Transonic Flow Through A compressor Blade Row, Using the Small-Disturbance Nonlinear Potential Equation", pp. 228-252 of <u>Transonic Flow Problems in Turbomachinery</u>, edited by T.C. Adamson and M.F. Platzer, Hemisphere Publishing Co., Washington, D.C. (1977).

4. Rae, W.J., "Calculations of Three-Dimensional Transonic Compressor Flowfields by a Relaxation Method", AIAA Paper 77-199, January 1977; published in the Journal of Energy, Vol. 1, No. 5, (September-October 1977), pp. 284-296.

5. Rae, W.J., "Computer Program for Relaxation Solutions of the Nonlinear Small-Disturbance Equations for Transonic Flow in An Axial Compressor Blade Row", AFOSR-TR-78-0855, AD-A053744 (April 1978).

used a relaxation method to solve the problem in the nonlinear small-disturbance approximation. Use of this approximation facilitated the adaptation of external-flow computational methods to the internal-flow case.

The present research program was undertaken with the aim of extending these numerical techniques, so as to handle more fully the nonlinearity of the problem. Thus, heavily loaded blades with large turning angles were considered, and the simplifications of small-disturbance theory (such as satisfaction of boundary conditions on mean-chord surfaces, neglect of trailing vortex-sheet deformation) were not used.

This Final Technical Report contains, in Sections 2 through 7, a summary of the significant accomplishments achieved in the performance of the research effort.

Section 2
OBJECTIVES

The goal of this program was to develop a numerical method for solving the equations of inviscid flow through a compressor blade row, in an approximation that retains the nonlinearity of the problem. Thus, blade rows with substantial pressure ratios and turning angles were to be considered. In addition, modifications of the flow field arising from highly deformed trailing vortex sheets were to be included.

The choice of a numerical method was left to be determined during the course of the investigation; the level of approximation intended for the research was that of the Euler equations or the full nonlinear potential equation. In the process of developing the numerical method and associated computer program, attention was to be given to methods for accelerating the rate of convergence of the calculations, and to the applicability of these methods for calculating unsteady flow patterns.

Section 3
FINAL STATUS OF THE RESEARCH

Details of the research done under this contract are presented in two AFOSR Scientific Reports (see References 6 and 7). This section contains a summary of those reports, plus other findings of the research that are not specifically recorded elsewhere.

The problem studied here is the inviscid flow through a blade row, assumed to be steady in a coordinate system fixed to the blades. Thus the a· ·ysis applies strictly to the situation where all other blade rows are many chord lengths away. Alternatively, for finite blade-row spacings, the analysis yields the steady component of the flow field. No further assumptions are made, however; the flow is fully three-dimensional and transonic, and the numerical solution method is capable of handling arbitrary shapes of the blades, hub, and shroud.

Since current transonic compressors operate with inlet relative Mach numbers that range up to around 1.4, it would be acceptable to make the further assumption of isentropic flow, and use the corresponding nonlinear potential-flow equation. At the beginning of this program, it was planned to adopt this approximation; however, further study of the numerical techniques available at that time led to a decision to use the full Euler equations, which would include nonisentropic flow. The principal reason for this decision was that the techniques then available were restricted to two-dimensional external flow, and the three-dimensional extensions of this work were not available at that time. In contrast, implicit time-marching methods had already been

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6. Rae, W.J., "A Computer Program for the Ives Transformation in Turbomachinery Cascades", Calspan Report No. 6275-A-3 (November 1980).

7. Rae, W.J., "An Application of Implicit Time Marching to Three Dimensional Flow Through a Compressor Blade Row", Calspan Report No. 6275-A-4 (November 1980).

published for three-dimensional, external-flow cases.[8-10] Thus many details about the solution algorithm were available, requiring a minimum number of adaptations for the internal-flow case.

During the last year of this program, several papers were published which contained extensions, to three dimensions, of approximate factorization techniques for solving the nonlinear potential equation.[11-13] A brief study of these extensions was made, including an adaptation to the case of internal flows. This work was carried up to the point of deriving the finite-difference equation and solution algorithm. However, no computer code was written for this method, since the scope of the program did not allow the pursuit of two parallel paths at that point. Details of the problem formulation are given in the Appendix, however, since they may be of further interest.

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8. Beam, R.M. and Warming, R.F., "An Implicit Finite-Difference Algorithm for Hyperbolic Systems in Conservation-Law Form", Journal of Computational Physics 22 (1976) pp. 87-110.

9. Steger, J.L., "Implicit Finite Difference Simulation of Flow About Arbitrary Geometrics With Application to Airfoils", AIAA Paper 77-665 (June 1977).

10. Kutler, P., Chakravarthy, S.R., and Lombard, C.P., "Supersonic Flow over Ablated Nose Tips Using An Unsteady Implicit Numerical Procedure", AIAA Paper 78-213 (January 1978).

11. Holst, T.L. and Ballhaus, W.F., "Fast Conservative Schemes for the Full Potential Equation Applied to Transonic Flows", AIAA Journal, Vol. 17, No. 2, (February 1979), pp. 145-152.

12. Holst, T.L. and Albert, J., "An Implicit Algorithm for the Conservative, Transonic Full Potential Equation with Effective Rotated Differencing", NASA TM 78570, (April 1979).

13. Holst, T.L., "A fast, Conservative Algorithm for Solving the Transonic Full Potential Equation", AIAA Paper 79-1456, (July 1979).

The development of the Euler-equation solver requried the formulation of several program elements: a means of handling the radius terms of the cylindrical coordinate system, derivation of a normal pressure gradient relation for use at solid boundaries, methods for enforcing the periodicity conditions, the boundary conditions, the mass-flow constraint, and the far-field conditions. Details about these program elements can be found in the AFOSR Scientific Report cited earlier (Reference 7).

More important than any of these items, however, was the choice of a grid-generation technique for transforming the flow field into a simple computational domain. After examining a number of options, a decision was made to use the conformal-transformation technique of Ives.[14] This technique had several advantages: first, the periodic boundaries are mapped into the interior of the computational domain, in such a way that the periodicity conditions on the flow field variables are satisfied automatically. Second, the blade surfaces are mapped into two of the boundaries in the transformed plane, while the regions at upstream and downstream infinity become a pair of points when transformed. And thirdly, the metrics of the coordinate transformation can be evaluated analytically, thus allowing accurate metric data even when a sparse grid is being used. Accordingly, a substantial effort was put into the development of a computer program for applying this transformation to cascades of rather arbitrary geometry. The results of this development are given in Reference 6.

A computer program incorporating all of these elements was then prepared, and was applied, on a very coarse grid, to the case of a two-dimensional cascade, for which nonlinear-potential solutions had been calculated in a previous effort at Calspan.[15] The remainder of the contract period was spent

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14. Ives, D.C. and Liutermoza, J.F., "Analysis of Transonic Cascade Flow Using Conformal Mapping and Relaxation Techniques", AIAA Journal <u>15</u> (1977), pp. 647-652.

15. Rae, W.J. and Homicz, G.F., "A Rectangular-Coordinate Method for Calculating Nonlinear Transonic Potential Flowfields in Compressor Cascades", AIAA Paper 78-248, (January 1978).

in debugging this program, and in making a lengthy series of modifications to it, in an attempt to achieve stable and accurate solutions. These were not ahcieved, due principally to singularities in the metric coefficients in the region near the trailing edges of the blades. The conformal transformation contains other singularities, which lie at upstream and downstream infinity; methods were developed, that were successful in eliminating the oscillations due to these singular points, but none of the approaches that were tried for the trailing-edge region gave adequate improvement. Further details about these approaches can be found in Reference 7.

Future developments of this program must start by solving the problem of the oscillations caused by the metric singularities at the trailing edge. It is not clear whether further modifications of the method now in use could remove the problem. An alternative is to replace the conformal mapping with an entirely different coordinate transformation. If that is done, certain elements in the computer program may need revision: for example, if the new transformation does not automatically satisfy the periodicity condition, some changes in the solution algorithm may be required.

It must also be pointed out that there are other problems which it was not possible to consider in depth because of the instabilities in the computer code. Among these are the means of accounting for deformed vortex sheets, the development of realistic far-field solutions and their coupling to the Kutta condition, and techniques for enforcing these results numerically. Lastly, there are opportunities for further progress with the nonlinear potential equation that should be pursued further.

Section 4
PUBLICATIONS

There have been no journal publications during this reporting period. The results of the research have been published in References 6 and 7.

Section 5
PERSONNEL

The principal investigator for this effort is Dr. William J. Rae. He has been assisted by Drs. J. C. Erickson, Jr., John A. Lordi, Gregory F. Homicz, and Joseph P. Nenni. On matters related to computer programming methods, he has had the assistance of Mr. John R. Moselle.

Section 6
INTERACTIONS

The principal investigator attended the following meetings:

1. ASME Gas Turbine Conference, London, England, April 9-13, 1978.

2. NASA Lewis Research Center, Workshop on Computational Fluid Dynamics Applied to Turbomachinery; Cleveland, OH, November 14-15, 1978.

3. NASA Lewis Research Center, Conference "Aeropropulsion 1979"; Cleveland, OH, May 15-16, 1979.

4. Air Force Aero Propulsion Laboratory, Wright-Patterson AFB, Ohio, for a meeting with Dr. A. Wennerstrom and Mr. M. Stibich, to discuss the work being done on this contract (June 22, 1979).

5. AIAA Short Course: "Effective Software Development for Aerodynamic Applications" given by W. F. Ballhaus, Jr., et al., Williamsburg, Virginia, July 21-22, 1979.

6. AIAA Computational Fluid Dynamics Conference, Williamsburg, Virginia, July 23-26, 1979.

Drs. James E. McCune and C. S. Tan of the MIT Gas Turbine Laboratory visited Calspan on January 14-15, 1980 for a discussion of their work on vorticity modeling, and Dr. Rae visited Dr. McCune on August 6 and 7, 1980, for further discussion.

Dr. Rae presented a seminar entitled "Computational Studies of Transonic Compressor Flow Fields", to the Department of Mechanical and Aerospace Engineering, State University of New York at Buffalo, November 6, 1980.

Copies of the quarterly progress reports describing the conformal transformation technique were sent to Dr. David C. Ives (Pratt & Whitney), Dr. Peter M. Sockol (NASA Lewis), Dr. W. Habashi (Concordia University, Montreal, Quebec), Dr. Richard A. Novak (General Electric Company, Lynn, Massachusetts) and to Professor Gino Moretti (Polytechnic of New York), and a copy of the computer deck for the conformal transformation method was sent to Dr. Sockol, Dr. Habashi, and Dr. Novak.

A copy of the computer deck from the previous AFOSR contract[5] was sent to Dr. David L. Whitfield (ARO, Inc., Tullahoma, Tennessee).

Section 7
INVENTIONS

There have been no inventions or patent disclosures stemming from this research effort.

APPENDIX

VELOCITY-POTENTIAL EQUATION

At the beginning of the present effort, the full nonlinear potential equation was envisioned as the level of approximation that would be used; but a review of the numerical techniques then available led to the conclusion that the Euler equations could be used more readily with the published algorithms. In contrast, solution methods for the three-dimensional potential equation were still under development. Accordingly, effort was devoted exclusively to applying the implicit time-marching technique.

However, major steps toward the required three-dimensional technique were published subsequently,[11-13] to the extent that it became feasible to undertake this approach. A limited study of these methods was made, including derivation of the appropriate partial differential equations and adaptation of the approximate-factorization algorithm to the special requirements of a transonic blade row. The resulting formulation of the problem is given in this Appendix. Preparation of a computer program to implement this method was not undertaken.

Partial Differential Equations

The basic equation is the continuity equation, written in the form:

$$\frac{\partial}{\partial z}(\sigma \bar{w}_z) + \frac{\partial}{\partial \rho}(\sigma \bar{w}_\rho) + \frac{1}{\rho}\frac{\partial}{\partial \theta}(\sigma \bar{w}_\theta) = -\frac{\sigma \bar{w}_\rho}{\rho} \qquad \text{(A-1)}$$

where steady flow has been assumed, and where dimensionless forms have been defined as follows, for the cylindrical coordinates x, r, θ the relative velocity components $w_{x,r,\theta}$, the density ρ and sound speed a:

$$z = \frac{\omega x}{U_\infty} \quad , \quad \rho = \frac{\omega r}{U_\infty} \quad , \quad \bar{W}_{z,\rho,\theta} = \frac{W_{x,r,\theta}}{U_\infty} \tag{A-2}$$

$$\sigma = \frac{\varrho}{\varrho_\infty} \quad , \quad A = \frac{a}{a_\infty}$$

The constant-rothalpy relation takes the form

$$\sigma^{\gamma-1} = A^2 = 1 - \frac{(\gamma-1)}{2}\left[M_\infty^2 - \frac{W^2}{a_\infty^2} + \frac{(\omega r)^2}{a_\infty^2}\right] \tag{A-3}$$

If a general coordinate transformation is now made:

$$\xi = \xi(z, \rho, \theta) \quad , \quad \eta = \eta(z, \rho, \theta) \quad , \quad \zeta = \zeta(z, \rho, \theta) \tag{A-4}$$

then the continuity equation can be written as

$$\frac{\partial}{\partial \xi}\left(\sigma \frac{U}{\mathcal{D}}\right) + \frac{\partial}{\partial \eta}\left(\sigma \frac{V}{\mathcal{D}}\right) + \frac{\partial}{\partial \zeta}\left(\sigma \frac{W}{\mathcal{D}}\right) = -\frac{\sigma \bar{W}_\rho}{\rho \mathcal{D}} \tag{A-5}$$

where

$$\mathcal{D} = \frac{\partial(\xi, \eta, \zeta)}{\partial(z, \rho, \theta)} = \begin{vmatrix} \xi_z & \xi_\rho & \xi_\theta \\ \eta_z & \eta_\rho & \eta_\theta \\ \zeta_z & \zeta_\rho & \zeta_\theta \end{vmatrix}$$

$$U = \xi_z \bar{W}_z + \xi_\rho \bar{W}_\rho + \frac{\xi_\theta}{\rho} \bar{W}_\theta$$

$$V = \eta_z \bar{W}_z + \eta_\rho \bar{W}_\rho + \frac{\eta_\theta}{\rho} \bar{W}_\theta$$

$$W = \zeta_z \bar{W}_z + \zeta_\rho \bar{W}_\rho + \frac{\zeta_\theta}{\rho} \bar{W}_\theta \tag{A-6}$$

These velocity components are then expressed in terms of the velocity potential. It is convenient to subtract the uniform helical approach flow, leaving the perturbation potential (which is not assumed to be small), i.e.:

$$\bar{W}_z = \frac{W_x}{U_\infty} = 1 + \frac{u}{U_\infty} = 1 + \phi_z$$

$$\bar{W}_\rho = \frac{W_r}{U_\infty} = \frac{v}{U_\infty} = \phi_\rho$$

$$\bar{W}_\theta = \frac{W_\theta}{U_\infty} = \frac{\omega r + w}{U_\infty} = \rho + \frac{w}{U_\infty} = \rho + \frac{\phi_\theta}{\rho} \tag{A-7}$$

These velocity-potential derivatives are then expressed in the ξ, η, ζ coordinates, giving:

$$\begin{aligned} U &= A_{01} + A_1\phi_\xi + A_4\phi_\eta + A_5\phi_\zeta \\ V &= A_{02} + A_4\phi_\xi + A_2\phi_\eta + A_6\phi_\zeta \\ W &= A_{03} + A_5\phi_\xi + A_6\phi_\eta + A_3\phi_\zeta \end{aligned} \tag{A-8}$$

where the A's are combinations of the metric coefficients

$$A_{01} = \xi_z + \xi_\theta \quad , \quad A_{02} = \eta_z + \eta_\theta \quad , \quad A_{03} = \zeta_z + \zeta_\theta$$

$$A_1 = \xi_z^2 + \xi_\rho^2 + \left(\frac{\xi_\theta}{\rho}\right)^2$$

$$A_2 = \eta_z^2 + \eta_\rho^2 + \left(\frac{\eta_\theta}{\rho}\right)^2$$

$$A_3 = \zeta_z^2 + \zeta_\rho^2 + \left(\frac{\zeta_\theta}{\rho}\right)^2$$

$$A_4 = \xi_z\eta_z + \xi_\rho\eta_\rho + \frac{\xi_\theta\eta_\theta}{\rho^2}$$

$$A_5 = \xi_z \zeta_z + \xi_\rho \zeta_\rho + \frac{\xi_\theta \zeta_\theta}{\rho^2}$$

$$A_6 = \eta_z \zeta_z + \eta_\rho \zeta_\rho + \frac{\eta_\theta \zeta_\theta}{\rho^2} \tag{A-9}$$

The finite-difference approximation to these equations (as developed in Refs. 11-13) includes two notable features: first, an approximate factorization of the finite-difference operator into the product of three one-dimensional operators, and second, the use of an upwind bias in the evaluation of the density ratio σ. The factorization of the operator is accomplished by replacing the original operator, L, (defined below) with a second operator, N. The operator L is defined as:

$$L\phi \equiv \overleftarrow{\delta}_\xi \left(\frac{\tilde{\sigma} U}{\mathcal{D}}\right)_{K+\frac{1}{2},N,L} + \overleftarrow{\delta}_\eta \left(\frac{\bar{\sigma} V}{\mathcal{D}}\right)_{K,N+\frac{1}{2},L} + \overleftarrow{\delta}_\zeta \left(\frac{\hat{\sigma} W}{\mathcal{D}}\right)_{K,N,L+\frac{1}{2}} + \frac{\sigma \tilde{W}_\rho}{\rho \mathcal{D}} = 0$$

where

$$\overleftarrow{\delta}_\xi \, (\,)_{K,L,N} \equiv \left[(\,)_{K,N,L} - (\,)_{K-1,N,L}\right] / \Delta\xi$$

with similar definitions of $\overleftarrow{\delta}_\eta$ and $\overleftarrow{\delta}_\zeta$. Here K,N,L denotes the grid-point indices in the ξ, η, and ζ directions. The three values of σ are evaluated with an upwind bias, by the formulas:

$$\tilde{\sigma}_{K+\frac{1}{2},N,L} = [(1-\nu)\sigma]_{K+\frac{1}{2},N,L} + \nu_{K+\frac{1}{2},N,L}\, \sigma_{K+\frac{1}{2}+r,N,L}$$

$$\bar{\sigma}_{K,N+\frac{1}{2},L} = [(1-\nu)\sigma]_{K,N+\frac{1}{2},L} + \nu_{K,N+\frac{1}{2},L}\, \sigma_{K,N+\frac{1}{2}+s,L}$$

$$\hat{\sigma}_{K,N,L+\frac{1}{2}} = [(1-\nu)\sigma]_{K,N,L+\frac{1}{2}} + \nu_{K,N,L+\frac{1}{2}}\, \sigma_{K,N,L+\frac{1}{2}+t} \tag{A-10}$$

where

$$
\begin{aligned}
r &= \mp 1 \quad \text{for} \quad U_{K+\frac{1}{2},N,L} \gtrless 0 \\
s &= \mp 1 \quad \text{for} \quad V_{K,N+\frac{1}{2},L} \gtrless 0 \\
t &= \mp 1 \quad \text{for} \quad W_{K,N,L+\frac{1}{2}} \gtrless 0
\end{aligned}
\tag{A-11}
$$

and where

$$
\begin{aligned}
\nu_{K+\frac{1}{2},N,L} &= max\,[(M^2_{K,N,L}-1)\cdot c,\,0\,] \quad \text{for } U > 0 \\
&= max\,[(M^2_{K+1,N,L}-1)\cdot c,\,0] \quad \text{for } U < 0
\end{aligned}
\tag{A-12}
$$

and with analogous definitions of $\nu_{K,N+\frac{1}{2},L}$ and $\nu_{K,N,L+\frac{1}{2}}$. The constant C is an input number between 1 and 2, of magnitude such that ν is less than 1 everywhere.

The replacement of operator L by operator N is done by writing the iteration process in the form:

$$
NC^{n} = -\omega L \phi^{n} \tag{A-13}
$$

where the superscript n denotes the iteration counter, ω is a relaxation factor, C is the correction to the solution:

$$
C^{n} = \phi^{n+1} - \phi^{n} \tag{A-14}
$$

and where the operator N is chosen as the product of three one-dimensional operators whose product closely matches the principal terms in L . For the present problem, the appropriate adaptation of Holst's formulas appears to be:

$$\alpha N C^{n}_{K,N,L} = -\left[\left(\alpha - \frac{1}{A_K}\vec{\delta}_\eta A_N \overleftarrow{\delta}_\eta\right)\left(A_K - \frac{1}{\alpha}\vec{\delta}_\zeta A_L \overleftarrow{\delta}_\zeta\right) - \alpha E_\xi^{+1} A_K\right]$$

$$\cdot\ (\alpha + \overleftarrow{\delta}_\xi)\, C_{K,N,L} \tag{A-15}$$

where α is a constant, which takes on a sequence of values as the iterations proceed, where the operator E_ξ^{+1} is defined as

$$E_\xi^{+1}\ (\ )_{K,N,L} = (\ )_{K+1,N,L} \tag{A-16}$$

and where the A's are related to the metric coefficients A_1, A_2, and A_3 :

$$A_K = \left(\frac{\tilde{\sigma} A_1}{\mathcal{D}}\right)^{n}_{K-\frac{1}{2},N,L}$$

$$A_N = \left(\frac{\bar{\sigma} A_2}{\mathcal{D}}\right)^{n}_{K,N-\frac{1}{2},L}$$

$$A_L = \left(\frac{\hat{\sigma} A_3}{\mathcal{D}}\right)^{n}_{K,N,L-\frac{1}{2}} \tag{A-17}$$

These operators are implemented by the following steps:

Step 1:

$$\left(\alpha - \frac{1}{A_K}\vec{\delta}_\eta A_N \overleftarrow{\delta}_\eta\right) g_{N,L} = \alpha\,\omega L \phi^{n}_{K,N,L} + \alpha A_{K+1} f_{K+1,N,L}$$

Step 2:

$$\left(A_K - \frac{1}{\alpha}\vec{\delta}_\zeta A_L \overleftarrow{\delta}_\zeta\right) f_{K,N,L} = g_{N,L}$$

Step 3:

$$(\alpha + \overleftarrow{\delta}_\xi)\, C_{K,N,L} = f_{K,N,L} \tag{A-18}$$

The sequence in which these steps are applied starts at the downstream edge of the grid, where K = KMX. For each N and L in this plane, Step 1 is applied, with $f_{KMX+1,N,L}$ arbitrarily set equal to zero. Next, $f_{KMX,N,L}$ is found by applying Step 2. Steps 1 and 2 are then repeated, for KMX-1, KMX-2, and so on. After all the K-planes have updated values of f, then Step 3 is applied, for K from 1 to KMX. Boundary values and periodicity conditions are then applied, explicitly, after which the sequence of Steps 1-3 is repeated.

These equations must be supplemented by specific means of enforcing the boundary, periodicity, inlet, and exit conditions.

REFERENCES

1. Rae, W.J., "Nonlinear Small-Disturbance Equations for Three-Dimensional Transonic Flow Through A Compressor Blade Row", Calspan Report No. AB-5487-A-1, AFOSR-TR-76-1082, AD-A031234, (August 1976).

2. Rae, W.J., "Relaxation Solutions for Three-Dimensional Transonic Flow Through a Compressor Blade Row, in the Nonlinear Small-Disturbance Approximation", Calspan Report No. AB-5487-A-2, AFOSR-TR-76-1081, AD-A032553, (August 1976).

3. Rae, W.J., "Finite-Difference Calculations of Three-Dimensional Transonic Flow Through A Compressor Blade Row, Using the Small-Disturbance Nonlinear Potential Equation", pp. 228-252 of Transonic Flow Problems in Turbomachinery, edited by T. C. Adamson and M. F. Platzer, Hemisphere Publishing Co., Washington, D. C. (1977).

4. Rae, W.J., "Calculations of Three-Dimensional Transonic Compressor Flowfields by a Relaxation Method", AIAA Paper 77-199, January 1977; published in the Journal of Energy, Vol. 1, No. 5, (September-October 1977), pp. 284-296.

5. Rae, W.J., "Computer Program for Relaxation Solutions of the Nonlinear Small-Disturbance Equations for Transonic Flow in An Axial Compressor Blade Row", AFOSR-TR-78-0855, AD-A053744, (April 1978).

6. Rae, W.J., "A Computer Program for the Ives Transformation in Turbomachinery Cascades", Calspan Report No. 6275-A-3, (November 1980).

7. Rae, W.J., "An Application of Implicit Time Marching to Three Dimensional Flow Through a Compressor Blade Row", Calspan Report No. 6275-A-4 (November 1980).

8. Beam, R.M. and Warming, R.F., "An Implicit Finite-Difference Algorithm for Hyperbolic Systems in Conservation-Law Form", Journal of Computational Physics, 22 (1976), pp. 87-110.

9. Steger, J.L., "Implicit Finite Difference Simulation of Flow About Arbitrary Geometrics with Application to Airfoils", AIAA Paper 77-665, (June 1977).

10. Kutler, P., Chakravarthy, S.R., and Lombard, C.P., "Supersonic Flow over Ablated Nose Tips Using an Unsteady Implicit Numerical Procedure", AIAA Paper 78-213, (January 1978).

11. Holst, T.L. and Ballhaus, W.F., "Fast Conservative Schemes for the Full Potential Equation Applied to Transonic Flows", AIAA Journal, Vol. 17, No. 2, (February 1979), pp. 145-152.

12. Holst, T.L. and Albert, J., "An Implicit Algorithm for the Conservative, Transonic Full Potential Equation with Effective Rotated Differencing", NASA TM 78570, (April 1979).

13. Holst, T.L., "A Fast, Conservative Algorithm for Solving the Transonic Full Potential Equation", AIAA Paper 79-1456, (July 1979).

14. Ives, D.C. and Liutermoza, J.F., "Analysis of Transonic Cascade Flow Using Conformal Mapping and Relaxation Techniques", AIAA Journal 15, (1977), pp. 647-652.

15. Rae, W.J., and Homicz, G.F., "A Rectangular-Coordinate Method for Calculating Nonlinear Transonic Potential Flowfields in Compressor Cascades", AIAA Paper 78-248, (January 1978).

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